

Mass spectrum of pentaquarks

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Abstract

We discuss the properties of the pentaquark in a collective stringlike model with a nonplanar configuration of the four quarks and the antiquark. In an application to the mass spectrum of exotic Θ baryons, we find that the ground state pentaquark has angular momentum and parity $J^P = 1/2^-$ and a small magnetic moment of $0.382 \mu_N$. The decay width is suppressed by the spatial overlap with the decay products.

1 Introduction

The building blocks of atomic nuclei, the nucleons, are composite extended objects, as is evident from the large anomalous magnetic moment, the excitation spectrum and the charge distribution of both the proton and the neutron. To first approximation, the internal structure of the nucleon at low energy can be ascribed to three bound constituent quarks q^3 . The nucleon and its excited states, collectively known as baryons, are accommodated into flavor singlets, octets and decuplets. The strangeness of the known baryons is either zero (nucleon, Δ) or negative (Λ , Σ , Ξ and Ω). Baryons with quantum numbers that cannot be obtained from triplets of quarks are called exotic.

Until recently, there was no experimental evidence for the existence of such exotic baryons. The discovery of the $\Theta(1540)$ baryon with positive strangeness $S = +1$ by the LEPS Collaboration [1] as the first example of an exotic baryon, has motivated an enormous amount of experimental and theoretical studies [2]. The width of this state is observed to be very small < 20 MeV (or perhaps as small as a few MeV's). More recently, the NA49 Collaboration [3] reported evidence for the existence of another exotic baryon $\Xi(1862)$ with strangeness $S = -2$. The Θ^+ and Ξ^{--} resonances are interpreted as $q^4\bar{q}$ pentaquarks belonging to a flavor antidecuplet with quark structure $uudd\bar{s}$ and $ddss\bar{u}$, respectively. In addition, there is now the first evidence [4] for a heavy pentaquark $\Theta_c(3099)$ in which the antistrange quark in the Θ^+ is replaced by an anticharm quark. The spin and parity of these states have not yet been determined experimentally. For a review of the experimental status we refer to [5].

Theoretical interpretations range from chiral soliton models [6] which provided the motivation for the experimental searches, correlated quark (or cluster) models [7], and various constituent quark models [8, 9]. A review of the theoretical literature of pentaquark models can be found in [10].

In this contribution, we discuss a collective stringlike model of $q^4\bar{q}$ pentaquarks in which the four quarks are located at the corners of an equilateral tetrahedron and the antiquark in its center. This nonplanar configuration arises as a consequence of the permutation symmetry of the four quarks [11]. As

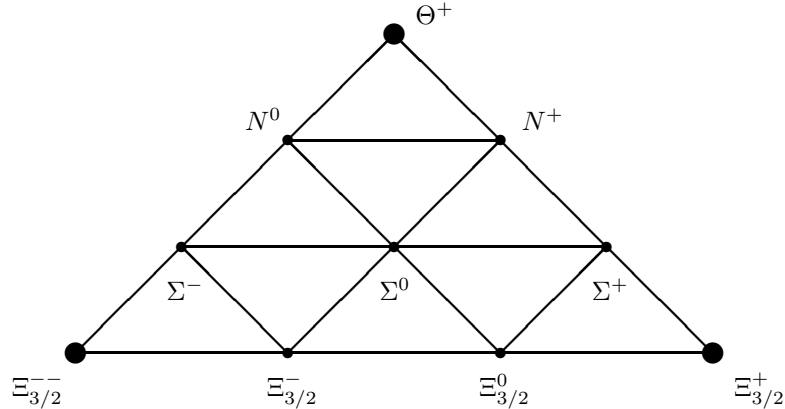


Figure 1: $SU(3)$ antidecuplet. The isospin-hypercharge multiplets are $(I, Y) = (0, 2)$, $(\frac{1}{2}, 1)$, $(1, 0)$ and $(\frac{3}{2}, -1)$. Exotic states are located at the corners and are indicated with \bullet .

an application, we discuss the spectrum of exotic Θ baryons, as well as the parity and magnetic moments of the ground state decuplet baryons. The decay width is suppressed by the spatial overlap with the decay products.

2 Pentaquark states

We consider pentaquarks to be built of five constituent parts whose dynamics is characterized by both internal and spatial degrees of freedom.

The internal degrees of freedom are: a flavor triplet for the quarks and a flavor anti-triplet for the antiquark (for the three light flavors: up, down and strange), a spin doublet (for spin $s = 1/2$) and a color triplet. The corresponding algebraic structure consists of the usual spin-flavor and color algebras $SU_{\text{sf}}(6) \otimes SU_c(3)$. The full decomposition of the spin-flavor states into spin and flavor states can be found in [9]

$$SU_{\text{sf}}(6) \supset SU_{\text{f}}(3) \otimes SU_{\text{s}}(2) \supset SU_{\text{I}}(2) \otimes U_{\text{Y}}(1) \otimes SU_{\text{s}}(2) . \quad (1)$$

The allowed flavor multiplets are singlets, octets, decuplets, antidecuplets, 27-plets and 35-plets. The first three have the same values of the isospin I and hypercharge Y as q^3 systems. However, the antidecuplets, the 27-plets and 35-plets contain exotic states which cannot be obtained from three-quark configurations. The latter states are more easily identified experimentally due to the uniqueness of their quantum numbers. The recently observed Θ^+ and Ξ^{--} resonances are interpreted as pentaquarks belonging to a flavor antidecuplet with isospin $I = 0$ and $I = 3/2$, respectively. In Fig. 1 the exotic states are indicated by a \bullet : the Θ^+ is an isosinglet $I = 0$ with hypercharge $Y = 2$ (strangeness $S = 1$), and the cascade pentaquarks $\Xi_{3/2}$ have hypercharge $Y = -1$ (strangeness $S = -2$) and isospin $I = 3/2$.

In the construction of the classification scheme we are guided by two conditions: the pentaquark wave function should be a color singlet and it should be antisymmetric under any permutation of the four quarks [9]. The permutation symmetry of the four-quark system is given by the permutation group S_4 which is isomorphic to the tetrahedral group \mathcal{T}_d . We use the labels of the latter to classify the states by their symmetry character: symmetric A_1 , antisymmetric A_2 or mixed symmetric E , F_2 or F_1 , corresponding to the Young tableaux [4], [1111], [22], [211] and [31], respectively.

The relative motion of the five constituent parts is described in terms of the Jacobi coordinates

$$\vec{\rho}_1 = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) ,$$

$$\begin{aligned}
\vec{\rho}_2 &= \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) , \\
\vec{\rho}_3 &= \frac{1}{\sqrt{12}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 - 3\vec{r}_4) , \\
\vec{\rho}_4 &= \frac{1}{\sqrt{20}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 - 4\vec{r}_5) ,
\end{aligned} \tag{2}$$

where \vec{r}_i ($i = 1, \dots, 4$) denote the coordinate of the i -th quark, and \vec{r}_5 that of the antiquark. The last Jacobi coordinate is symmetric under the interchange of the quark coordinates, and hence transforms as A_1 under \mathcal{T}_d , whereas the first three transform as three components of F_2 .

The total pentaquark wave function is the product of the spin, flavor, color and orbital wave functions. Since the color part of the pentaquark wave function is a singlet and that of the antiquark an anti-triplet, the color wave function of the four-quark configuration is a triplet with F_1 symmetry. The total q^4 wave function is antisymmetric (A_2), hence the orbital-spin-flavor part has to have F_2 symmetry

$$\psi = [\psi_{F_1}^c \times \psi_{F_2}^{\text{osf}}]_{A_2} . \tag{3}$$

Here the square brackets $[\dots]$ denote the tensor coupling under the tetrahedral group \mathcal{T}_d .

3 Stringlike model

In this section, we discuss a stringlike model for pentaquarks, which is a generalization of a collective stringlike model developed for q^3 baryons [12]. The general approach is that of introducing an interacting boson model to describe the orbital excitations of the pentaquark. We introduce a dipole boson with $L^p = 1^-$ for each independent relative coordinate, and an auxiliary scalar boson with $L^p = 0^+$, which leads to a compact spectrum-generating algebra of $U(13)$ for the radial excitations. As a consequence of the invariance of the interactions under the permutation symmetry of the four quarks, the most favorable geometric configuration is an equilateral tetrahedron in which the four quarks are located at the four corners and the antiquark in its center [11] (see Fig. 2). This configuration was also considered in [13] in which arguments based on the flux-tube model were used to suggest a nonplanar structure for the $\Theta(1540)$ pentaquark to explain its narrow width. In the flux-tube model, the strong color field between a pair of a quark and an antiquark forms a flux tube which confines them. For the pentaquark there would be four such flux tubes connecting the quarks with the antiquark.

3.1 Mass spectrum of Θ baryons

Hadronic spectra are characterized by the occurrence of linear Regge trajectories with almost identical slopes for baryons and mesons. Such a behavior is also expected on basis of soft QCD strings in which the strings elongate as they rotate. In the same spirit as for algebraic models of stringlike q^3 baryons [12], we use the mass-squared operator

$$M^2 = M_0^2 + M_{\text{vib}}^2 + M_{\text{rot}}^2 + M_{\text{sf}}^2 . \tag{4}$$

The vibrational term M_{vib}^2 describes the vibrational spectrum corresponding to the normal modes of a tetrahedral $q^4\bar{q}$ configuration

$$M_{\text{vib}}^2 = \epsilon_1 \nu_1 + \epsilon_2 (\nu_{2a} + \nu_{2b}) + \epsilon_3 (\nu_{3a} + \nu_{3b} + \nu_{3c}) + \epsilon_4 (\nu_{4a} + \nu_{4b} + \nu_{4c}) . \tag{5}$$

The rotational energies are given by a term linear in the orbital angular momentum L which is responsible for the linear Regge trajectories in baryon and meson spectra

$$M_{\text{rot}}^2 = \alpha L . \tag{6}$$

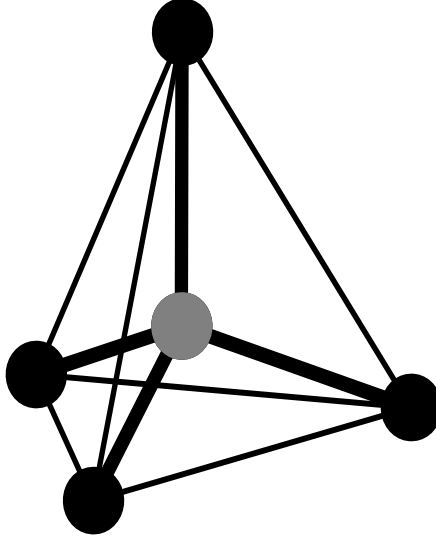


Figure 2: Geometry of stringlike pentaquarks

The spin-flavor part is expressed in a Gürsey-Radicati form, i.e. in terms of Casimir invariants of the spin-flavor groups of Eq. (1)

$$M_{\text{sf}}^2 = a C_{2SU_{\text{sf}}(6)} + b C_{2SU_{\text{f}}(3)} + c C_{2SU_{\text{s}}(2)} + d C_{1U_{\text{Y}}(1)} + e C_{1U_{\text{Y}}(1)}^2 + f C_{2SU_{\text{f}}(2)} . \quad (7)$$

The coefficients α , a , b , c , d , e and f are taken from a previous study of the nonstrange and strange baryon resonances [12], and the constant M_0^2 is determined by identifying the ground state exotic pentaquark with the recently observed $\Theta(1540)$ resonance. Since the lowest orbital states with $L^P = 0^+$ and 1^- are interpreted as rotational states, for these excitations there is no contribution from the vibrational terms ϵ_1 , ϵ_2 , ϵ_3 and ϵ_4 . The results for the lowest Θ pentaquarks (with strangeness $S = +1$) are shown in Fig. 3.

The lowest pentaquark belongs to the flavor antidecuplet with spin $s = 1/2$ and isospin $I = 0$, in agreement with the available experimental information which indicates that the $\Theta(1540)$ is an isosinglet. In the present calculation, the ground state pentaquark belongs to the $[42111]_{F_2}$ spin-flavor multiplet, indicated in Fig. 3 by its dimension 1134, and an orbital excitation 0^+ with A_1 symmetry. Therefore, the ground state has angular momentum and parity $J^P = 1/2^-$, in agreement with recent work on QCD sum rules [14] and lattice QCD [15], but contrary to the chiral soliton model [6], various cluster models [7] and a lattice calculation [16] that predict a ground state with positive parity. The first excited state at 1599 MeV is an isospin triplet Θ_1 -state of the 27-plet with the same value of angular momentum and parity $J^P = 1/2^-$. The lowest pentaquark state with positive parity occurs at 1668 MeV and belongs to the $[51111]_{A_1}$ spin-flavor multiplet (with dimension 700) and an orbital excitation 1^- with F_2 symmetry. In the absence of a spin-orbit coupling, in this case we have a doublet with angular momentum and parity $J^P = 1/2^+, 3/2^+$.

There is some preliminary evidence from the CLAS Collaboration for the existence of two peaks in the nK^+ invariant mass distribution at 1523 and 1573 MeV [17]. The mass difference between these two peaks is very close to the mass difference in the stringlike model between the ground state pentaquark at 1540 MeV (fitted) and the first excited state Θ_1 at 1599 MeV.

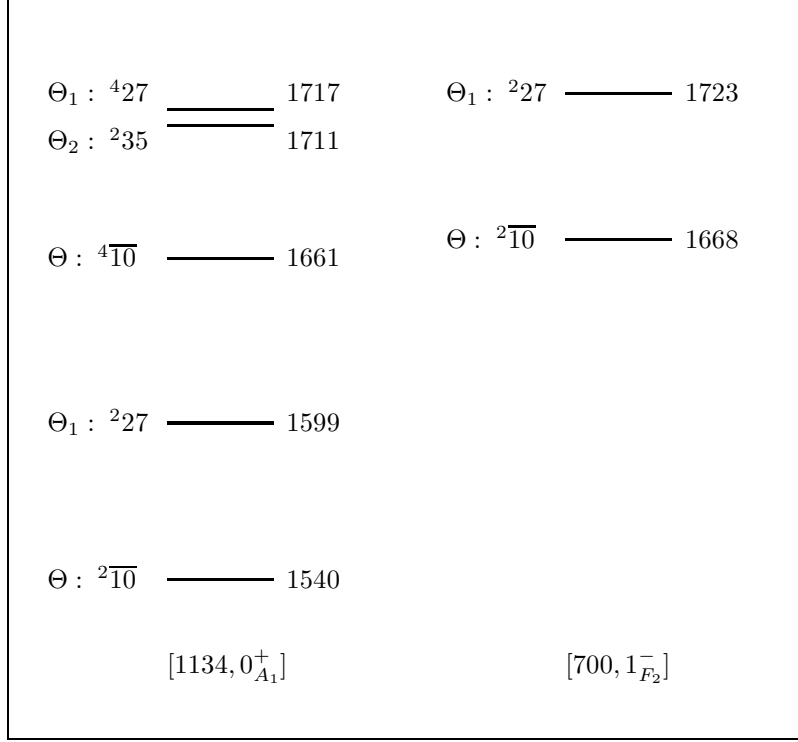


Figure 3: Spectrum of Θ pentaquarks. Masses are given in MeV.

3.2 Magnetic moments

The magnetic moment of a multiquark system is given by the sum of the magnetic moments of its constituent parts

$$\vec{\mu} = \vec{\mu}_{\text{spin}} + \vec{\mu}_{\text{orb}} = \sum_i \mu_i (2\vec{s}_i + \vec{\ell}_i) , \quad (8)$$

where the quark magnetic moments μ_u , μ_d and μ_s are determined from the proton, neutron and Λ magnetic moments and satisfy $\mu_q = -\mu_{\bar{q}}$.

The $SU_{\text{sf}}(6)$ wave function of the ground state pentaquark has the general structure

$$\psi_{A_2} = \left[\psi_{F_1}^c \times \left[\psi_{A_1}^o \times \psi_{F_2}^{\text{sf}} \right]_{F_2} \right]_{A_2} . \quad (9)$$

Since the ground state orbital wave function has $L^p = 0^+$, the magnetic moment only depends on the spin part. For the Θ^+ exotic state we obtain

$$\mu_{\Theta^+} = (2\mu_u + 2\mu_d + \mu_s)/3 = 0.382 \mu_N , \quad (10)$$

in agreement with the result obtained [18] for the MIT bag model. These results for the magnetic moments are independent of the orbital wave functions, and are valid for any quark model in which the eigenstates have good $SU_{\text{sf}}(6)$ spin-flavor symmetry.

Table 1: Magnetic moments of the ground state antidecuplet pentaquarks for $J^P = \frac{1}{2}^-$ in μ_N .

	μ_{th}	μ_{calc}
Θ^+	$(6\mu_u + 6\mu_d + 3\mu_s)/9$	0.382
N^0	$(5\mu_u + 6\mu_d + 4\mu_s)/9$	0.108
N^+	$(6\mu_u + 5\mu_d + 4\mu_s)/9$	0.422
Σ^-	$(4\mu_u + 6\mu_d + 5\mu_s)/9$	-0.166
Σ^0	$(5\mu_u + 5\mu_d + 5\mu_s)/9$	0.148
Σ^+	$(6\mu_u + 4\mu_d + 5\mu_s)/9$	0.462
$\Xi_{3/2}^{--}$	$(3\mu_u + 6\mu_d + 6\mu_s)/9$	-0.440
$\Xi_{3/2}^-$	$(4\mu_u + 5\mu_d + 6\mu_s)/9$	-0.126
$\Xi_{3/2}^0$	$(5\mu_u + 4\mu_d + 6\mu_s)/9$	0.188
$\Xi_{3/2}^+$	$(6\mu_u + 3\mu_d + 6\mu_s)/9$	0.502

In Table 1, we present the magnetic moments of all members of the ground state antidecuplet. The magnetic moments are typically an order of magnitude smaller than the proton magnetic moment. In addition, they satisfy the generalized Coleman-Glashow sum rules [19, 20] for the antidecuplet

$$\begin{aligned}
\mu_{\Theta^+} + \mu_{\Xi_{3/2}^+} &= \mu_{N^+} + \mu_{\Sigma^+} , \\
\mu_{\Theta^+} + \mu_{\Xi_{3/2}^{--}} &= \mu_{N^0} + \mu_{\Sigma^-} , \\
\mu_{\Xi_{3/2}^{--}} + \mu_{\Xi_{3/2}^+} &= \mu_{\Xi_{3/2}^-} + \mu_{\Xi_{3/2}^0} ,
\end{aligned} \tag{11}$$

and

$$2\mu_{\Sigma^0} = \mu_{\Sigma^-} + \mu_{\Sigma^+} = \mu_{N^0} + \mu_{\Xi_{3/2}^0} = \mu_{N^+} + \mu_{\Xi_{3/2}^-} . \tag{12}$$

The same sum rules hold for the chiral quark-soliton model in the chiral limit [21]. In the limit of equal quark masses $m_u = m_d = m_s = m$, the magnetic moments of the antidecuplet pentaquark states become proportional to the electric charges $\mu_i = Q_i/18m$, which implies that the sum of the magnetic moments of all members of the antidecuplet vanishes $\sum_i \mu_i = 0$.

4 Summary and conclusions

In this contribution, we have discussed a stringlike model of pentaquarks, in which the four quarks are located at the corners of an equilateral tetrahedron with the antiquark in its center. Geometrically this is the most stable configuration. The ground state pentaquark belongs to the flavor antidecuplet, has angular momentum and parity $J^P = 1/2^-$ and, in comparison with the proton, has a small magnetic moment. The width is expected to be narrow due to a large suppression in the spatial overlap between the pentaquark and its decay products [13].

The first report of the discovery of the pentaquark has triggered an enormous amount of experimental and theoretical studies of the properties of exotic baryons. Nevertheless, there still exist many doubts and questions about the existence of this state, since in addition to various confirmations there are also

several experiments in which no signal has been observed [22]. Hence, it is of the utmost importance to understand the origin between these apparently contradictory results, and to have irrefutable proof for or against its existence [5]. If confirmed, the measurement of the quantum numbers of the $\Theta(1540)$, especially the angular momentum and parity, and the excited pentaquark states, may help to distinguish between different models and to gain more insight into the relevant degrees of freedom and the underlying dynamics that determines the properties of exotic baryons.

Acknowledgements

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